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Evaluating the solvency capital requirement of interest rate risk in Solvency II



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ABSTRACT

In this paper, the question addressed is as to whether the Solvency II standard formula provides a good measure for the interest rate risk an insurer is facing. In order to answer this question, several simplifications of the standard formula are considered and an alternative method is proposed to simulate the future term structures of interest rates to provide a better insight in the interest rate risk of an insurer. This method makes use of cubic Hermite spline interpolation up to the last liquid point of the term structure and the Smith-Wilson extrapolation for interest rates beyond this point. The method is able to produce a wide range of term structure shapes, and provides a good fit with the yield curve and one year stress scenarios from the standard formula. By means of a case study, three illustrative liability portfolios are considered and the Solvency Capital Requirement (SCR) of interest rate risk is calculated based on the standard formula and the alternative method. From the case study it is concluded that the simplifications in the standard formula can lead to serious drawbacks in the management of interest rate risk, especially with respect of liabilities with high expected premium income, long term guarantees and/or a material risk margin.

1. INTRODUCTION

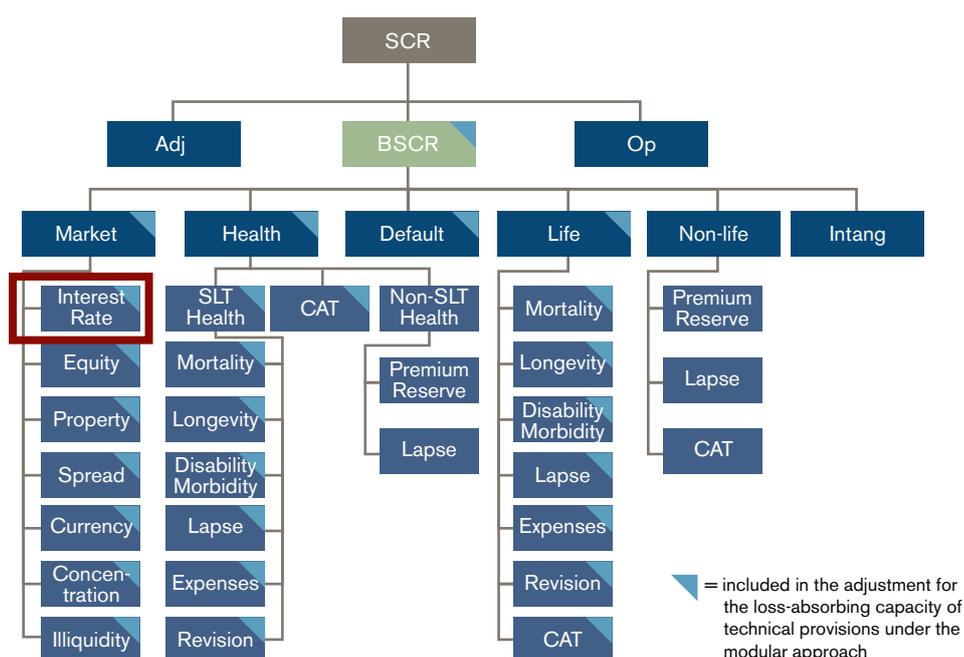
On 1 January 2014, the new Solvency II regime is expected to be introduced across Europe. It will replace the Solvency I requirements imposed in 1970 and will apply to all insurers with an annual gross premium income in excess of €5 million or gross technical provisions exceeding €25 million. The aim of Solvency II is to achieve consistency across Europe employing market consistent balance sheets, risk-based capital, own risk and solvency assessment (ORSA), senior management accountability and supervisory assessment.

One aspect of the Solvency II regulations is to ensure that insurers hold adequate risk-based capital. This capital is represented by the Solvency Capital Requirement (SCR). The SCR should correspond to the one-year 99.5% Value-at-Risk of the basic own funds of an insurance or reinsurance undertaking.

The SCR under the standard formula approach is derived by aggregating capital requirements specified in several sub-modules, as is illustrated in Figure 1, corresponding to the different risks the insurer can face. One of these sub-modules corresponds to interest rate risk. In the standard formula, Solvency II will prescribe a set of stress scenarios in order to derive the SCR for interest rate risk. Although the specifications for Solvency II have not been finalized, the framework used to calculate the SCR is presented, tested and evaluated in so-called Quantitative Impact Studies (QIS), of which the latest one is QIS5. This impact study was performed in 2010 using 31 December 2009 as the reporting date.

In this paper, we analyse whether the methodology as prescribed in the standard formula provides a good measure for the interest rate risk an insurer is facing. This question is particularly relevant since insurance companies may opt for a (partial) internal model to determine their SCR instead of using the standard formula.

FIGURE 1: THE AGGREGATION SCHEDULE USED TO DERIVE THE SCR IN QIS5 BY AGGREGATING ALL UNDERLYING SUB-MODULES. THE INTEREST RATE RISK IS PART OF THE MARKET RISK MODULE.



Most of the numerical examples presented in this paper are based on the QIS5 technical specifications. However, we believe that given the publication of the draft Level 2 implementing measures at the end of 2011 and the current discussions on the standard formula (particularly the definition of the construction of the relevant yield curve to be used to derive the technical provisions), the analysis is still very much up to date.

This paper is structured as follows. Chapter 2 provides some background concepts on the valuation methods that are applied in Solvency II and the definition of the yield curve. Chapter 3 outlines the standard formula for interest rate risk and discusses some of the simplifications of this method. In Chapter 4 we present an alternative method of deriving the required capital for interest rate risk. Based on a case study, the alternative method is compared with the standard formula in Chapters 5 and 6. Finally, based on the results from the case study, some conclusions are presented.

2. MARKET-CONSISTENT VALUATION OF LIABILITIES IN SOLVENCY II

The valuation of the assets and liabilities, as set out in Article 75 of the Framework Solvency II Directive, requires an economic, market-consistent approach. This means that a large part of the insurer's balance sheet (both assets and liabilities) is exposed to interest rates and hence are subject to changes in the yield curve.

Since the valuation of the assets is mostly done on a mark-to-market value, the focus of this analysis will be on the valuation and risk of the liabilities. Usually, the market-consistent value of the technical provisions of the insurer is split into two parts: the best estimate and the risk margin.

The best estimate can be defined as the probability weighted average of discounted future cash flows. The risk margin is calculated using the cost of capital method, and equals the discounted cost of providing an amount of required own funds equal to the SCR for non-hedgeable risks necessary to support fluctuations in the cash flows over the whole time horizon.

Both the best estimate and the risk margin are therefore affected by the term structure of interest rates. The yield curve that needs to be used in the valuation of liabilities will be prescribed and consists of two parts. For maturities up to the last liquid point, the curve is equal to the Inter-bank swap curve corresponding to the market the insurer is in. The last liquid point represents the longest maturity for which interest rates can be observed in liquid markets. For maturities beyond this last liquid point, the yield curve is extrapolated using the Smith-Wilson extrapolation technique (2001).

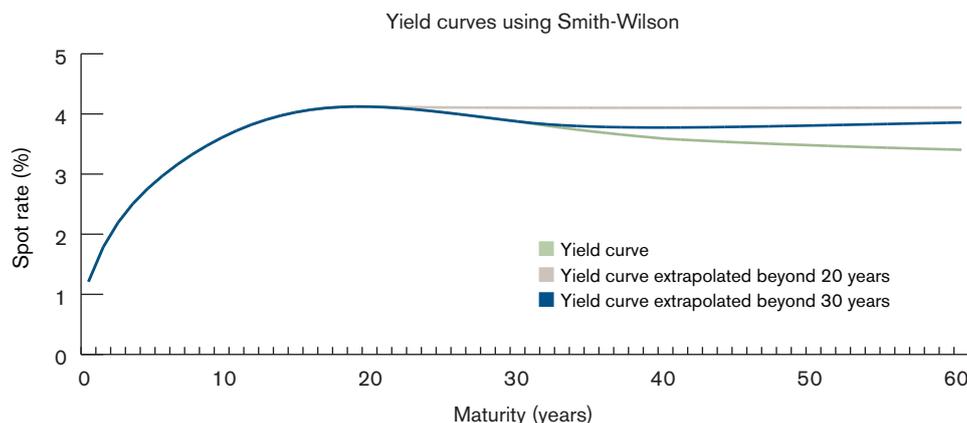
The inputs for the Smith-Wilson extrapolation are the yield curve prior to the last liquid point, the unconditional ultimate forward rate (UFR) and the speed of transition.

The UFR is defined as the rate all term structures of interest rates will eventually converge to and is assumed to be constant over time. A macro-economic assessment has been applied in order to derive its value for different currencies. For the Eurozone, the UFR is equal to 4.2%, which is the sum of the expected annual inflation rate (2%) and the expected annual short-term return on risk-free bonds (2.2%).

The speed of transition represents the time it will take the Smith-Wilson curve to converge to the UFR.

In Figure 2, the yield curve based on the Euro Inter-bank swap rates up to maturities of 60 years is presented. This yield curve changes when the Smith-Wilson extrapolation technique is applied. The figure also shows the impact of the maturity that is considered to be the last liquid point.

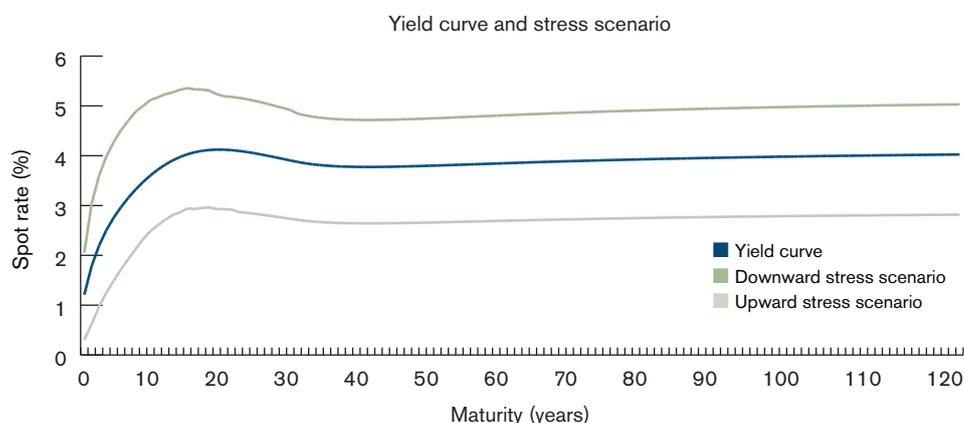
FIGURE 2: THE YIELD CURVE AND CORRESPONDING YIELD CURVE EXTRAPOLATED USING A LAST LIQUID POINT OF 20 AND 30 YEARS, AND UFR OF 4.2%. THE YIELD CURVE SHOWN IN THIS FIGURE CORRESPONDS TO THE EURO ZERO RATES EXCLUDING LIQUIDITY PREMIUM AS AT 31 DECEMBER 2009.



3. THE SCR OF INTEREST RATE RISK IN THE STANDARD FORMULA

Interest rate risk exists for all assets and liabilities for which the net asset value is sensitive to changes in the term structure of interest rates or interest rate volatility. The calculations of capital requirements in the interest rate risk module are based on specified scenarios. These scenarios are defined by a downward and upward stress of the term structure of interest rates. An example of these scenarios is shown in Figure 3.

FIGURE 3: THE EURO YIELD CURVE AND THE CORRESPONDING ONE-YEAR 99.5% VAR INTEREST RATE STRESS SCENARIOS USED IN THE QIS5 EXERCISE (2010).



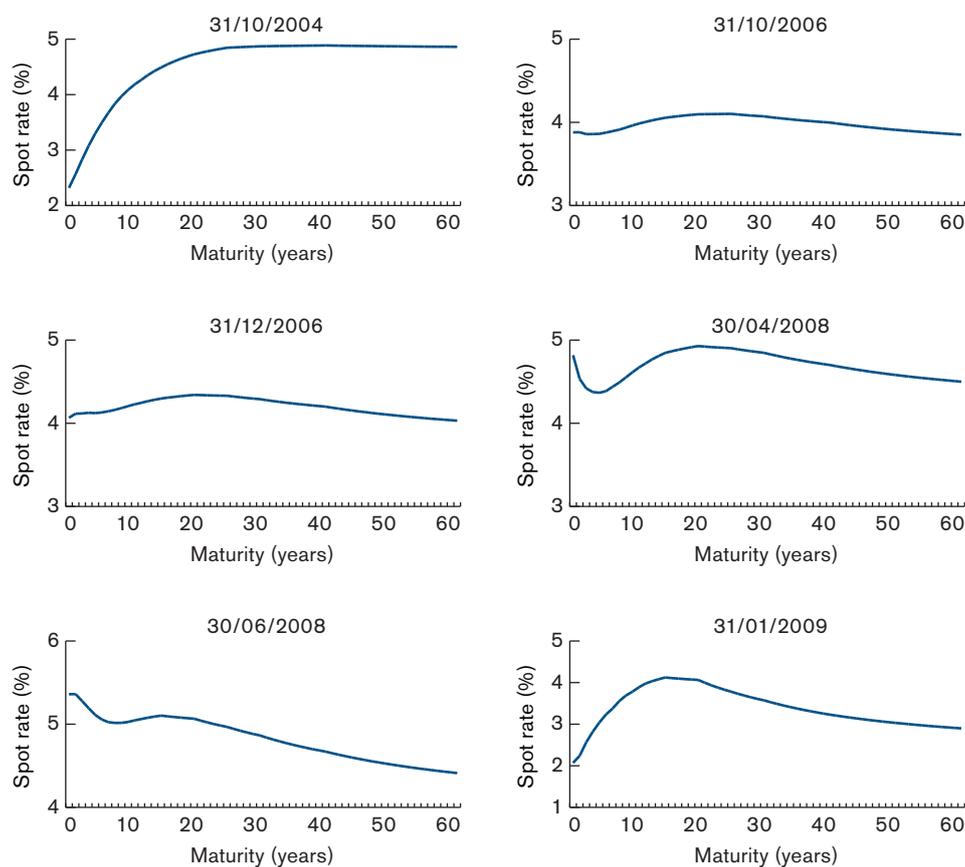
In order to derive the capital requirement, the assets and liabilities are revalued using the yield curves from the two stress scenarios. The maximum expected negative change in net asset value from these two stresses is then used as the required capital for interest rate risk.

A number of simplifications have been applied to define the stress scenarios for the interest rate risk module in the standard formula. In this paper, we outline three of these simplifications and propose an alternative method to demonstrate the possible impact of these simplifications.

The first simplification is the use of only two near parallel scenarios. We acknowledge that the two stress scenarios are derived using a principal component analysis (explaining 99.98% of the variability of the annual percentage interest rate change in each of the maturities in the underlying datasets¹) of historical term structure data. However, the components resulting from this analysis could be used to construct non parallel scenarios. Aggregating these components in only two near parallel stress scenarios is clearly a simplification since, in practice, we have (even recently) observed that non-parallel movements in the term structure of interest rates are also feasible (for examples see Figure 4).

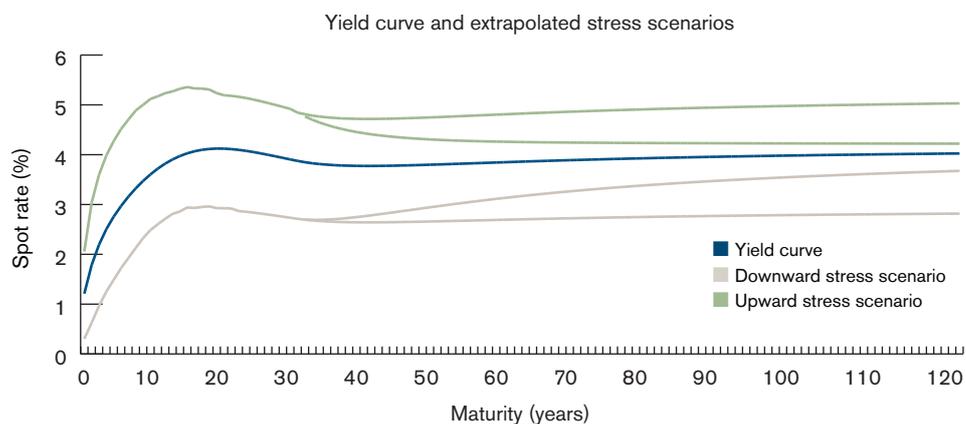
¹ QIS5 Calibration Paper (2010), p. 28.

FIGURE 4: HISTORICAL YIELD CURVES AT DIFFERENT REPORTING DATES.



The second simplification we would like to address is the stress applied to the term structure after the assumed last liquid point. As one can see from Figure 3, upward and downward shocks remain at the same level beyond the last liquid point. This however, contradicts the use of the UFR. If we assume that the UFR will be applied at the same level after a one year time period, the yield curve in a stressed scenario will also converge to the UFR. It would therefore be reasonable for the interest rate stress scenarios to converge to the UFR instead of leaving them constant beyond the last liquid point. This is illustrated in Figure 5 on page 8.

FIGURE 5: EXAMPLE OF APPLYING THE SMITH-WILSON EXTRAPOLATION TO THE STRESS SCENARIOS FOR THE EURO YIELD CURVE AT THE LAST LIQUID POINT (TERM 30) TO CONVERGE TO THE UFR OF 4.2%.



The third simplification covered in this paper is that only the best estimate is taken into account when revaluing the liabilities in the stress scenarios and not the risk margin as well. However, as shown in the previous chapter, the value of the risk margin is clearly dependent on the yield curve, both through the projection of the required capital for non-hedgeable risks as well as the discounting of the future capital charges. Where the risk margin is a material part of the technical provisions, not taking into account the risk margin in the derivation of the required capital for interest rate risk can potentially be a serious drawback of the standard formula.

4. AN ALTERNATIVE METHOD FOR DERIVING THE REQUIRED CAPITAL FOR INTEREST RATE RISK

In this chapter, an alternative method to derive the required capital for interest rate risk using simulations of future term structures of interest rates is presented. The alternative method to derive the required capital for interest rate risk consists of the following steps:

1. Simulate a set of possible spot rates over a one year time horizon up to the last liquid point.
2. Simulate the term structure of interest rates for all maturities up to the last liquid point using a cubic Hermite spline interpolation function.
3. Extrapolate the term structures after the last liquid point using the Smith-Wilson technique.
4. For each simulation revalue all assets and liabilities, including the risk margin, to derive the distribution of the change in net asset value. From this distribution, the corresponding 99.5% percentile and thus the required capital for interest rate risk can be derived.

Regarding the distribution of spot rates, Brigo and Mercurio (2006) state that the normal distribution or the log-normal distribution are commonly used in practice to assume a distribution on the spot rates. We have therefore assumed the spot rates in the alternative method to be distributed normally.

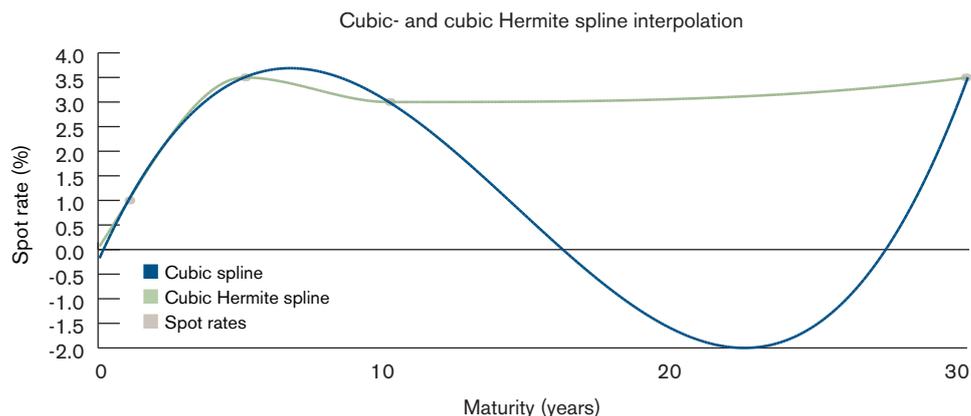
The method used for the interpolation of the spot rates must provide enough flexibility to generate all possible realistic term structure shapes. For example, it must at least be able to generate historical observed yield curves as presented in Figure 4.

We have interpolated the spot rates using cubic Hermite spline interpolation as presented in Fritsch and Carlson (1980). While this method is commonly used in physics, it is not widely used in the insurance industry. The method interpolates each set of neighbouring spot rates with a linear combination of Hermite functions, i.e., a special case of 'flattened' fourth-degree polynomials. By using a linear combination of the Hermite functions, the smoothness of the overall interpolated term structure is assured and realistic term structure shapes can be constructed.

Other interpolation methods we have considered are cubic spline interpolation, the Nelson-Siegel method (1987) and the Svensson method (1997). While these methods are widely used in the industry, all of these methods have limitations that are restrictive for the application of deriving a required capital for interest rate risk.

Cubic spline interpolation is not suitable since it produces oscillation as can be seen in Figure 7. Therefore, realistic term structure shapes are not guaranteed.

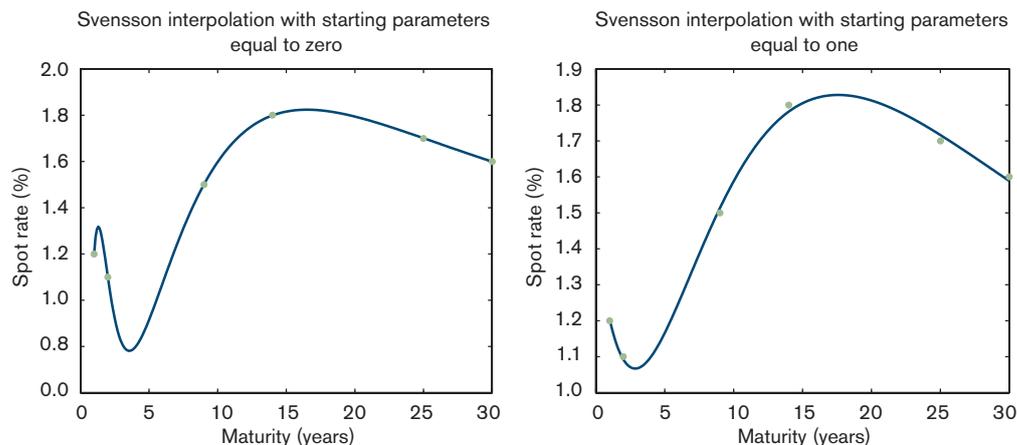
FIGURE 6: CONSTRUCTION OF A TERM STRUCTURE BY INTERPOLATING A SIMULATED SET OF SPOT RATES USING BOTH CUBIC SPLINE INTERPOLATION AND CUBIC HERMITE SPLINE INTERPOLATION. FROM THIS FIGURE IT IMMEDIATELY BECOMES CLEAR THAT THE USE OF A HERMITE SPLINE IN ORDER TO INTERPOLATE THE SPOT RATES PREVENTS OSCILLATION THAT IS PRESENT WHEN USING CUBIC SPLINE INTERPOLATION.



The Nelson-Siegel method is not flexible enough to construct the required range of term structure shapes since it makes use of a function that is only able to interpolate between three spot rates².

Finally, the Svensson method is able to construct realistic term structure shapes. However, it is not suitable for simulating multiple interest rate shocks since, for the same set of spot rates, using different set of starting values, the fit of the term structures can vary (see Figure 7).

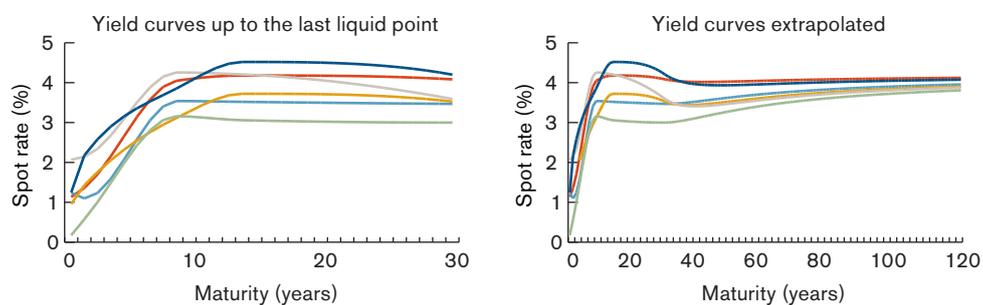
FIGURE 7: SPOT RATES INTERPOLATED USING SVENSSON METHOD WITH TWO DIFFERENT STARTING VALUES USED IN THE SEARCH ALGORITHM.



² The original Nelson-Siegel (1987) function has four parameters that are used to fit the term structure of interest rates. Diebold and Li (2006), however, argue that one of these parameters can be fixed.

For the extrapolation of the spot rates beyond the last liquid point, we use the Smith-Wilson extrapolation technique, because we assume that following years' yield curve used for the valuation of liabilities will again converge to the UFR. Examples of term structure shocks that can be constructed following the three steps outlined above are shown in Figure 8.

FIGURE 8 LEFT: TERM STRUCTURE SHOCKS SIMULATED AND INTERPOLATED USING CUBIC HERMITE SPLINE INTERPOLATION. RIGHT: THE SIMULATED TERM STRUCTURE SHOCKS EXTRAPOLATED USING SMITH-WILSON.



5. CASE STUDY: SIMULATING TERM STRUCTURES

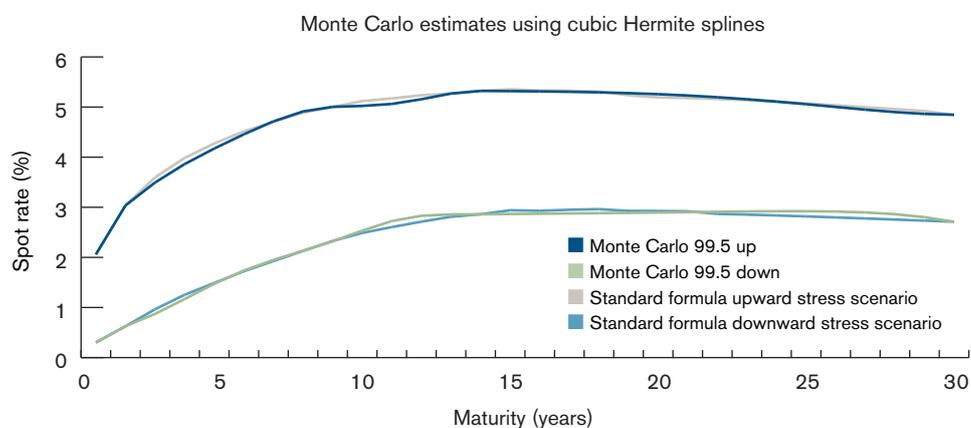
Now that an alternative method of constructing a distribution of realistic term structure shocks is defined, we wish to compare the impact of this alternative method to the standard formula using a case study.

In order to get a fair comparison between the approach in the standard formula and our alternative method, we have calibrated the parameters in the interest rate method to match the two interest rate scenarios of the standard formula as closely as possible. In the case study, we have used the QIS5 technical specifications (2010) as an example to represent the standard formula.

We have calibrated the mean and standard deviation of five spot rates with different maturities based on the upward and downward stress scenarios. In Figure 9, the result of the calibration of these spot rates is shown (using 100,000 simulations and cubic Hermite spline interpolation for the remaining spot rates).

To simulate realistic term structures, we have assumed a correlation surface between the five different spot rates. This assumption enables the method to simulate term structures that are non-parallel to the yield curve.

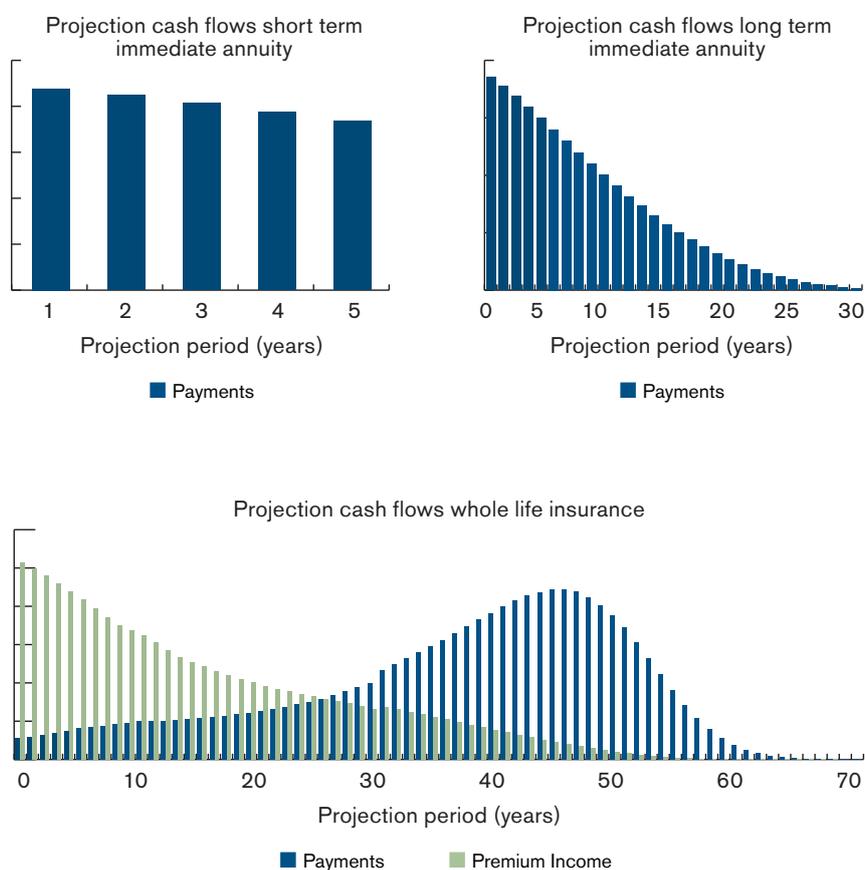
FIGURE 9: REPRESENTATION OF THE 99.5 MONTE CARLO PERCENTILES RESULTING FROM THE ALTERNATIVE METHOD IN COMBINATION WITH THE 99.5 STANDARD FORMULA PERCENTILES.



6. CASE STUDY: EVALUATING INTEREST RATE RISK IN SOLVENCY II

The standard formula and the alternative method are evaluated using three different types of liability products: short-term immediate annuities, long-term immediate annuities and whole life insurance policies. The patterns of the present value best estimate cash flows are presented in Figure 10.

FIGURE 10: PATTERNS OF THE PRESENT VALUE BEST ESTIMATE CASH FLOWS CORRESPONDING TO THE SHORT-TERM IMMEDIATE ANNUITY, LONG-TERM IMMEDIATE ANNUITY AND WHOLE LIFE INSURANCE.



First, we evaluate the standard formula and the alternative method based on the best estimate liabilities. Afterwards we show the impact of taking into account the risk margin in the alternative method.

The results of the comparison of the standard formula and the alternative method applied to the three products are included in Table 1. The standard formula's and alternative method's scenarios corresponding to these results can be found in Figure 11.

TABLE 1: RESULTS OF THE STANDARD FORMULA AND ALTERNATIVE METHOD APPLIED TO THE THREE PRODUCTS. THE NUMBERS SHOWN ARE PERCENTAGES IN TERMS OF THE BEST ESTIMATE LIABILITY VALUE ON THE BALANCE SHEET.

	STANDARD FORMULA	ALTERNATIVE METHOD
SHORT-TERM IMMEDIATE ANNUITY	3.6%	3.8%
LONG-TERM IMMEDIATE ANNUITY	10.2%	8.7%
WHOLE LIFE INSURANCE	44.3%	60.7%

When considering the short-term immediate annuity portfolio, there is no material difference in the required capital between the two methods. This is mainly due to the small duration of the cash-flow pattern.

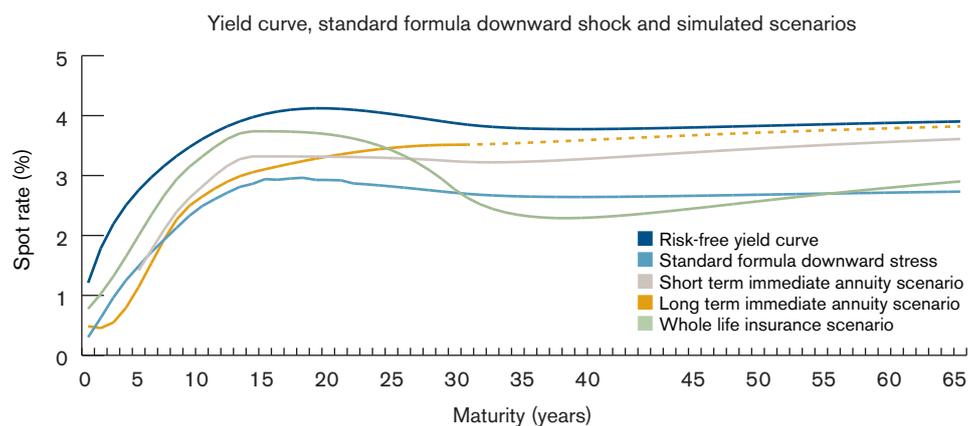
For the long-term immediate annuity portfolio, the results are as follows. There turns out to be only a small difference between the two methods. The difference is caused by the simulated 99.5% scenario being on average less severe than the standard formula due to the introduction of correlation in the alternative method, as can be seen in Figure 11. Using non-parallel movements of the yield curve, the probability that all spot rates on the yield curve will decrease simultaneously becomes less than one, resulting in a decrease of the required capital. This effect becomes apparent when long-term cash flows are part of the portfolio.

In case of the whole life insurance, the majority of the premium income occurs at the beginning, whereas the majority of the payments occur between 30 and 60 years. This is an important observation when considering the Smith-Wilson extrapolation which is applied in the alternative method.

From Figure 11, it follows that the simulated scenario that corresponds to the whole life insurance is more severe than the 99.5% standard formula downward shock between the maturities of 30 and 60 years and less severe for the other maturities. Combining this with the shape of the cash-flow pattern explains that the required capital using the alternative method is materially higher than in case of the standard formula.

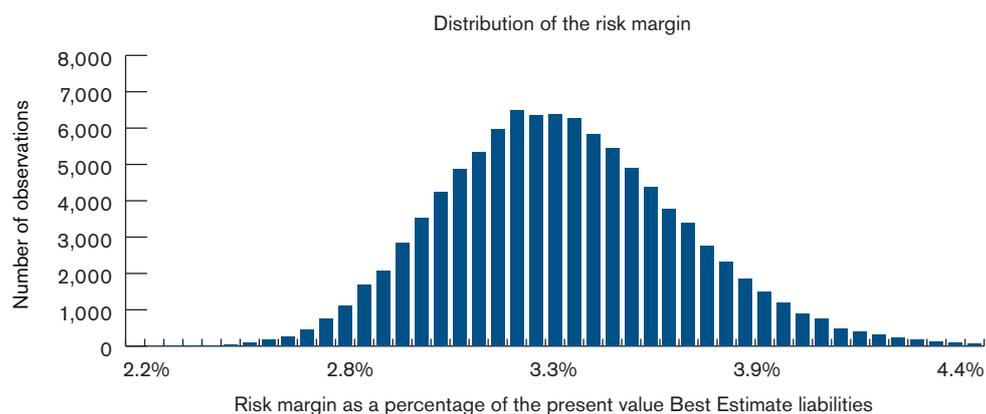
The shape of the yield curve after 30 years is determined by the Smith-Wilson extrapolation. Although the UFR is set at a higher rate than the 30-year simulated spot rate, the yield curve is still decreasing the first nine maturities after the last liquid point. This is caused by shape of the yield curve up to the last liquid point and the use of the Smith-Wilson extrapolation technique. Since the yield curve is decreasing after maturities of 15 years and the Smith-Wilson extrapolation technique assures a smooth convergence to the UFR beyond the last liquid point, the yield curve decreases before converging to the UFR of 4.2%.

FIGURE 11: ILLUSTRATION OF THE EURO YIELD CURVE, 99.5% STANDARD FORMULA DOWNWARD SHOCK AND THE SIMULATED SCENARIOS CORRESPONDING TO THE REQUIRED CAPITALS IN THE ALTERNATIVE METHOD. IN CASE OF THE SIMULATED SCENARIOS, THE SOLID PARTS REPRESENT THE INTEREST RATES INFLUENCING THE REQUIRED CAPITAL. THE DOTTED PARTS REPRESENT THE REMAINING PART OF THE CURVE.



In order to determine whether taking into account the risk margin has an impact on the required capital, we have revalued the risk margin of the long-term immediate annuity portfolio using the simulated term structures. The distribution of the risk margin based on the scenarios is shown in Figure 12.

FIGURE 12: DISTRIBUTION OF THE RISK MARGIN IN PERCENTAGES OF THE PRESENT VALUE BEST ESTIMATE LIABILITIES OF THE LONG-TERM IMMEDIATE ANNUITY, WHEN IT IS SUBJECT TO THE SIMULATED SCENARIOS RESULTING FROM THE ALTERNATIVE METHOD.



From the distribution it is clear that the risk margin is dependent on the interest rate. Where the risk margin is equal to 3.5% of the best estimate in the base scenario, the risk margin increases with 31% to 4.6% in the 1 in 200 years scenario. As a result, taking the risk margin into account in the derivation of the required capital of interest rate risk increases this capital with 12%.

CONCLUSION

In the previous chapters, we considered the standard formula for the required capital for interest rate risk. We determined that the standard formula is using several simplifications and proposed an alternative method where future term structures of interest rates were simulated. This method makes use of a normal distribution of the individual spot rates, a correlation structure, cubic Hermite spline interpolation up to the last liquid point of the term structure and the Smith-Wilson extrapolation for interest rates beyond this point.

Based on the case study, we conclude that the alternative method is able to simulate realistic term structures and provides a good fit with the yield curve and one-year stress scenarios in the standard formula. This is useful because it provides the opportunity to use the alternative method consistently with the assumptions in the standard formula.

Using three types of liability portfolios, we also showed that for some simple products, the simplifications in the standard formula are not necessarily limiting. When considering more complex products, the simplifications can potentially lead to more drawbacks in the management of interest rate risk.

If premium income is material, the shape of the yield curve is an important aspect of the interest rate risk. The use of near-parallel scenarios in the standard formula neglects this risk.

In case of guarantees after the last liquid point, the use of the UFR in constructing the yield curve has a material impact on the market-consistent value of the technical provisions. When we assume that this UFR will also be in place in a stress scenario, the use of the UFR is important to consider when deriving the required capital for interest rate risk. Taking the UFR into account in the alternative method leads to material differences from the standard formula results.

We also showed that if the risk margin is a material part of the technical provision, leaving out the impact of the change in risk margin from the analysis is a serious drawback.

Using the alternative method is not only useful for determining required capital but is also vital in understanding the dependency of interest rates to the valuation of the portfolio.

REFERENCES

- [1] Brigo, D. and Mercurio F. (2006). *Interest Rate Models: Theory and Practice—With Smile, Inflation and Credit*. New York, NY: Springer Verlag.
- [2] CEIOPS. (2010). QIS5 Technical Specifications. *European Commissions—Internal Market and Services DG*.
- [3] CEIOPS. (2010). QIS5 Calibration Paper.
- [4] Diebold, F.X. and Li, C. (2006). Forecasting the Term Structure of Government Bond Yields. *Journal of Econometrics*, Volume 130, Issue 2: pp. 337 – 364.
- [5] Fritsch, F.N. and Carlson, R.E. (1980). Monotone Piecewise Cubic Interpolation. *SIAM Journal on Numerical Analysis*, 17 : pp. 238 – 246.
- [6] Nelson, C.R. and Siegel, A.F. (1987). Parsimonious Modeling of Yield Curves. *The Journal of Business*, Volume 60, Issue 4: pp. 473 – 489.
- [7] Smith, A. and Wilson, T. (2001). Fitting Yield Curves with Long Term Constraints. *Research Notes, Bacon and Woodrow*.
- [8] Svensson, L.E.O. (1994). Estimating and Interpreting Forward Interest Rates: Sweden 1992 – 1994. *NBER working paper No. 4871*.



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